REAL OPTION VALUE, CH 13

REAL REVENUE OPTIONS

March 2015

A provider of perishable goods or services with or without some pricing power (prices may be endogenous or exogenous) may hold several real revenue options. (I) Revenue options may be written or purchased calls, written or purchased puts, with different payoffs for the service provider and the customer. (II) There may be different prices for different types of customers (sometimes for slightly different products, or at different times and places), and opportunities to switch customers. (III) The service provider may allocate constrained supplies to market segments. (IV) The facility operator may sell more future services than feasible capacity, assuming some purchasers with refundable (or no advanced) payments will cancel.

From the prospective of the producer of the perishable good or service, there are four basic types of payoffs from arrangements with the prospective customer. Each arrangement offers advantages and disadvantages for the seller (service provider) and the buyer (customer), and results in a distinct strategy payoff for the provider that resembles one of the characteristic option payoff strategies. For convenience assume that the seller creates part of the perceived value of the future good or service by committing to a specific known cost, which may be a mixture of capacity investment cost (such as a stadium, hotel, scheduled airline service or event) and committed operating costs (such as operating fees, employee and semi-durable costs). For instance, an airline may offer a scheduled service between Denver and Phoenix, which involves prior costs of the airport slots and fees, the airplane and fixed costs, plus the costs of the fuel, labor and other expenses during the flight. Assume the airline cannot cancel the flight if sufficient passengers do not show up.

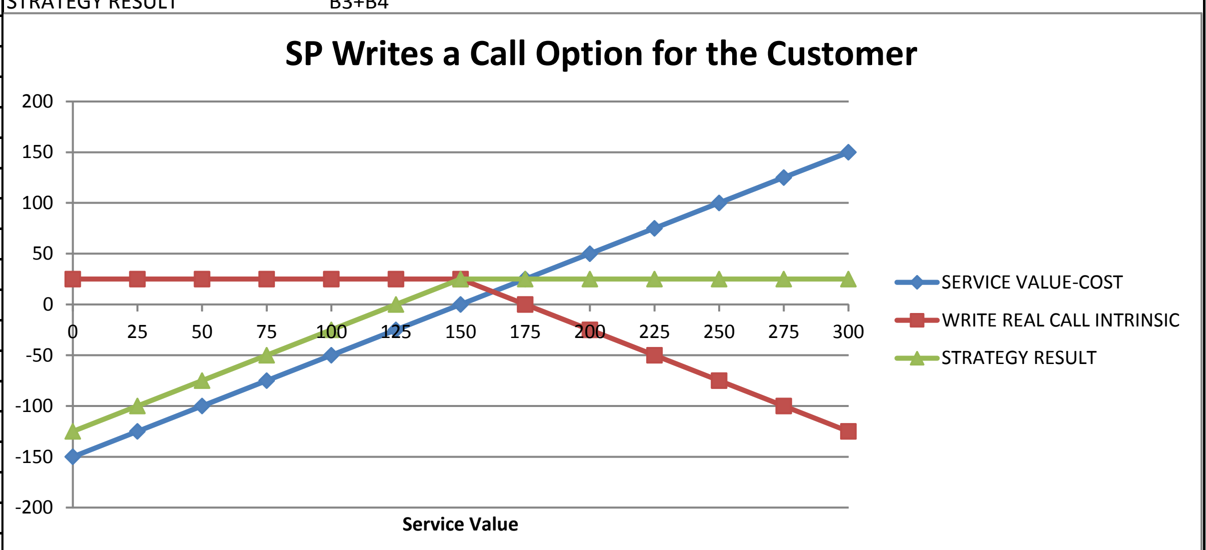
The next section provides some basic payoff diagrams for a service provider like an airline, which is more profitable the higher the revenue (seats sold times ticket price), entering into various arrangements with customers, which alter the profits as revenue changes. Section two suggests a formal model for switching between different market segments, such as upper and discount class fares for an airline. Section three looks at allocating limited capacity between different market segments. The last section examines overbooking options, where some passengers especially those with refundable tickets do not show up for scheduled flights, and overbooking allows the service provider to fill perishable service capacity. All of these activities are typical alternative actions in the tourism, accommodation and transportation industries.

I Revenue Arrangements and Resulting STRATEGY

SELLER (SERVICE PROVIDER) SELLS A CALL OPTION TO THE CUSTOMER: STRATEGY ONE

For various reasons, as a service to the customer, who values the security of a reservation, but is uncertain about using it, or because the airline may want to partially fund the venture, the airline sells refundable tickets. The option premium is the credit availability and no interest on the advanced sales amount, and the option element is the refund for no shows. The customer pays $$ for the option to purchase the specific flight and does not receive any refund upon flying, but %$$ if not flying. The option premium should be higher, the greater the value the customer puts on a guaranteed seat combined with a greater uncertainty on using that reservation. The higher the apparent load factor prior to departure, the greater the option premium, while customers will pay little for flights that have high apparent capacity, or low load factors. Few reserve seats in nearly empty churches.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**REAL OPTION PAYOFFS**													
2	SERVICE VALUE	0	25	50	75	100	125	150	175	200	225	250	275	300
3	SERVICE VALUE-COST	-150	-125	-100	-75	-50	-25	0	25	50	75	100	125	150
4	WRITE REAL CALL INTRINSIC	25	25	25	25	25	25	25	0	-25	-50	-75	-100	-125
5	STRATEGY RESULT	-125	-100	-75	-50	-25	0	25	25	25	25	25	25	25
6	Value of Commited Cost, K	150												
7	Option cost, y	25												
8	**Own a valuable service which involves a pre-commited cost K, sell to customer an option to buy the service at nil cost, for immediate payment of option premium of y.**													
9	**STRATEGY RESULT equals write real put on net service value.**													
10	WRITE REAL CALL INTRINSIC	IF(B3>0,-B3+$B$7,$B$7)												
11	STRATEGY RESULT	B3+B4												

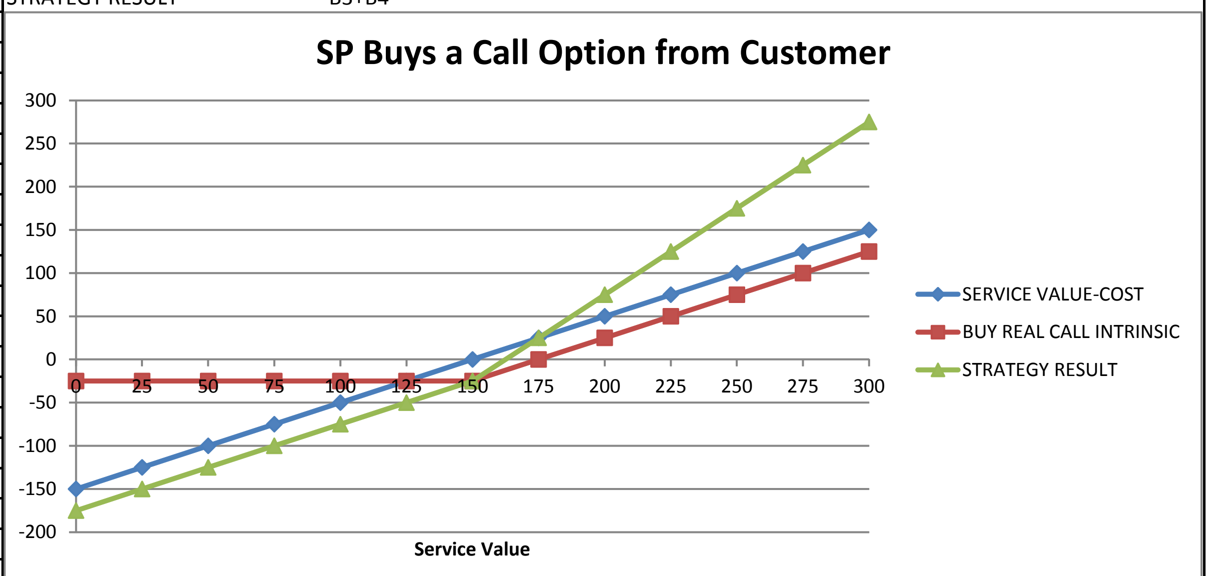


30	Option cost, y	25

For writing revenue call options, the seller receives an equivalent option premium, sometimes upfront, for alternative future actions that the buyer may take, sometimes at specified exercise prices or limits. Sometimes the apparent option premium is observable as the difference between truly non-refundable and completely refundable air ticket reservations. Here is an illustration where the seller (such an airline) sells a call option to the passenger at a refundable fare which is 25 higher than a non-refundable fare. If the value of service offered increases, or the passenger departs, the airline does not receive any additional fare, but otherwise there are empty seats. The present value of the scheduled flight costs is 150 per available seat. If the current value of one seat is 150, with an extra cost of a refundable fair of 25, the resulting strategy is a profit for the airline of 25. But if the "service value" falls to 0, the airline has to provide the service, but refundable passengers will not show up, resulting in a loss for the airline of 125. The net strategy result for the service provider is similar to <u>writing a real put option</u>, suffering on the downside, with the upside limited to the option premium.

SELLER BUYS A CALL OPTION FROM THE CUSTOMER: STRATEGY TWO

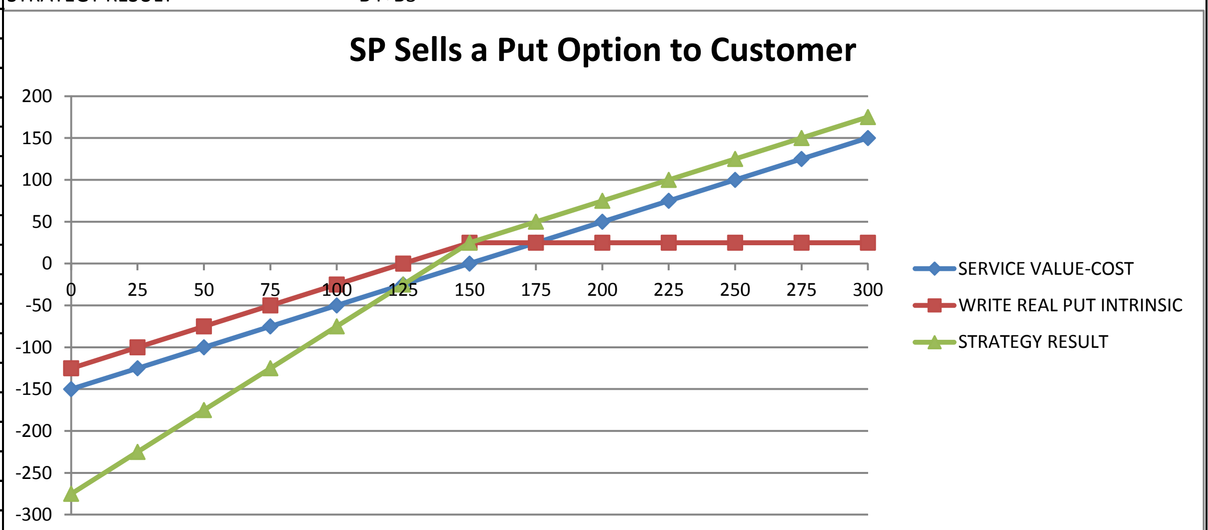
	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**REAL OPTION PAYOFFS**													
2	SERVICE VALUE	0	25	50	75	100	125	150	175	200	225	250	275	300
3	SERVICE VALUE-COST	-150	-125	-100	-75	-50	-25	0	25	50	75	100	125	150
4	BUY REAL CALL INTRINSIC	-25	-25	-25	-25	-25	-25	-25	0	25	50	75	100	125
5	STRATEGY RESULT	-175	-150	-125	-100	-75	-50	-25	25	75	125	175	225	275
6	Value of Commited Cost, K	150												
7	Option cost, y	25												
8	**Own a valuable service which involves a pre-commited cost K, buy from customer an option to repurchase the right to the service for immediate payment of option premium of y.**													
9	**STRATEGY RESULT equals write real put on net service value.**													
10	BUY REAL CALL INTRINSIC	IF(B3>0,B3-$B$7,-$B$7)												
11	STRATEGY RESULT	B3+B4												
31	Option cost, y	25												



In buying revenue call options from customers, the seller pays an option premium usually in terms of reduced prices, or alternatively a rebate to customers. In overbooking, an airline obtains the possibility of repurchasing tickets in order to re-sell to other customers willing to pay higher prices, assuming that the original customer does not value the specific date and flight reserved as much as the compensation. In overbooking compensation, airlines induce passengers to withdraw. These arrangements are not unusual in the energy world, where some firms agree to interruptible energy from an energy supplier. The strategy result is similar to holding a super call option which is equivalent to a leveraged operation, where the potential upside is enhanced, but the downside is increased by the option premium.

PROVIDER SELLS A PUT OPTION TO THE CUSTOMER: STRATEGY THREE

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**REAL OPTION PAYOFFS**													
2	SERVICE VALUE	0	25	50	75	100	125	150	175	200	225	250	275	300
3	SERVICE VALUE-COST	-150	-125	-100	-75	-50	-25	0	25	50	75	100	125	150
4	WRITE REAL PUT INTRINSIC	-125	-100	-75	-50	-25	0	25	25	25	25	25	25	25
5	STRATEGY RESULT	-275	-225	-175	-125	-75	-25	25	50	75	100	125	150	175
6	Value of Commited Cost, K	150												
7	Option cost, y	25												
8	**Sell a valuable service which involves a pre-commited cost K & sell to the customer an option to resell the service back to the producer**													
9	**The seller receives from the customer an upfront premium, or alternatively requires the customer to pay a higher initial price.**													
10	**STRATEGY RESULT equals write a super put on service value.**													
11	WRITE REAL PUT INTRINSIC	IF(B3<0,B3+$B$7,$B$7)												
12	STRATEGY RESULT	B4+B3												



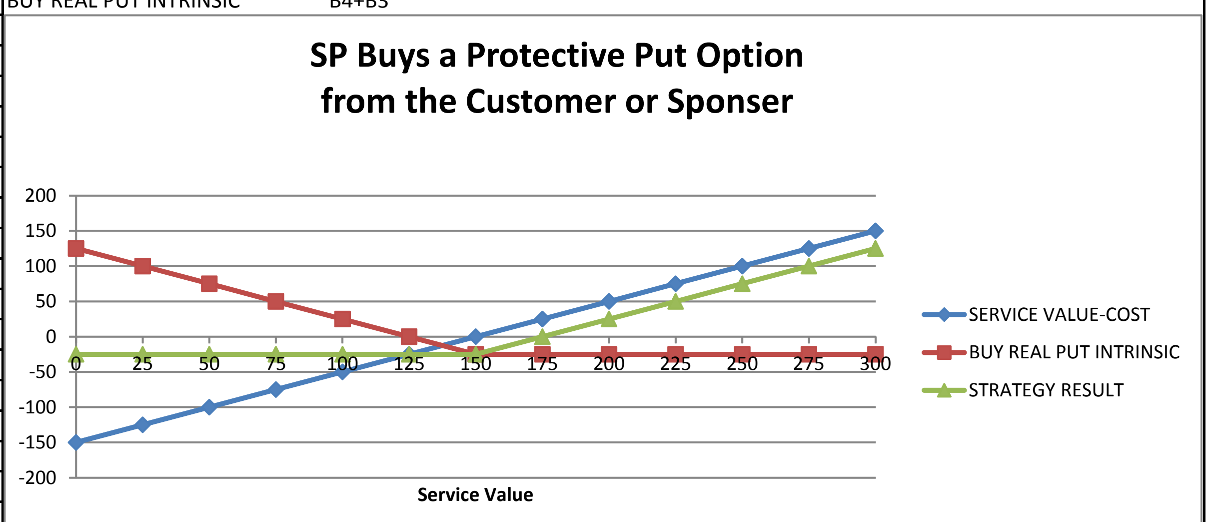
	A	B
31	Option cost, y	25

If the provider offers to repurchase tickets from the customers if required, also known as "money back guarantees", the provider has in effect written a revenue put option. Sometimes this is presented as a quality assurance scheme. The net strategy amounts to the provider holding a super put option, equivalent to leveraged operations, where the downside is increased, but the upside is increased only by the option premium. The service provider benefits from upward movements in the service value, but loses more on the downside, than without such money-back guarantees.

PROVIDER BUYS PUT OPTIONS FROM THE CUSTOMER: STRATEGY FOUR

In this case, the seller of the good or services requires the customer to purchase the service at a fixed price in case the price falls. An example might be a developer (provider of accommodation services) of an educational establishment which an institution decides will enhance the community. But if not enough external students show up, that institution agrees to take up the places, possibly at reduced rates. The take-or-pay guarantee from the institution provides the assurance for external funding for the development. Sometimes these guarantees are public and explicit, sometimes covert. The net strategy results in the service provider holding a net call option on the service provided, with limited downside risk and the upside reduced by the option premium.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**REAL OPTION PAYOFFS**													
2	SERVICE VALUE	0	25	50	75	100	125	150	175	200	225	250	275	300
3	SERVICE VALUE-COST	-150	-125	-100	-75	-50	-25	0	25	50	75	100	125	150
4	BUY REAL PUT INTRINSIC	125	100	75	50	25	0	-25	-25	-25	-25	-25	-25	-25
5	STRATEGY RESULT	-25	-25	-25	-25	-25	-25	-25	0	25	50	75	100	125
6	Value of Commited Cost, K	150												
7	Option cost, y	25												
8	**Own a valuable service which involves a pre-commited cost K, buy from the customer an option to sell the service at**													
9	**the commited cost K. The seller pays the customer an upfront premium, or alternatively grants the customer a reduced K.**													
10	**STRATEGY RESULT equals buy real call on net service value at nil exercise cost.**													
11	SERVICE VALUE-COST	IF(B3<0,-B3-$B$7,-$B$7)												
12	BUY REAL PUT INTRINSIC	B4+B3												
30	Option cost, y	25												



Note that often service providers might purchase or sell options on required inputs such as aircraft and facilities, wages and fuel. Providers **buy calls on inputs** such as supplementary aircraft or employees when **e**xisting aircraft fail to operate, or additional facilities are required due to massive overbooking. Car rental firms often acquire additional cars from other firms when there is a geographical imbalance, or renters do not return cars when scheduled. Providers may **sell calls on inputs** such as supplying spare available capacity to other service providers, and may **buy puts on inputs** or **sell puts on inputs**. The strategy result for the service provider will typically be the opposite of writing/buying options on their outputs.

II REVENUE SWITCHING USE OPTIONS

Revenue switching options are often embedded in facilities, or situations, sometimes developed with the imagination and initiative of the participant, or manager. For instance, hotel managers may have the option of reconfiguring rooms into "luxury" or "budget", airlines separate discount from first (or upper) class, and universities may offer premier MBA classes at a higher tuition rate than standard Masters classes. Sometimes the higher fare or rate for luxury-first-premier is due partially to higher operating costs and services [more space for air travellers], but it may be accompanied by lower volumes or alternatively simply higher margins. Many articles on pricing and revenue management are focused on such price differentiation, assuming that prices are endogenous and demand is stochastic.

For convenience, assume initially that separate prices for upper/discount classes are exogenous and stochastic, and may be correlated. The facility manager decides periodically what differential prices justify reconfiguring a facility at a deterministic or constant switching cost based on a perpetual output switching model that allows for differential operating costs. As a simplification (and reduction in the number of equations required for a solution), a single switch, such as from discount to a permanent upper class only, is also considered. The model for switching to the best of two outputs is adapted from Dockendorf and Paxson (2013).

1 OUTPUT SWITCHING OPTIONS

When is the right time for an operator of a flexible facility such as an airline to switch back and forth between two possible outputs like upper/discount classes in order to maximise value when switching costs are taken into account? Which factors should be monitored in making these decisions? How much should an investor pay for such a flexible operating asset? What are the strategy implications for the operator, investor and possibly for policy makers?

The traditional approach to determine switching boundaries between two operating modes is to discount future cash flows and use Jevons-Marshallian present value triggers. This methodology does not fully capture the option value which may arise due to the uncertainty in future output prices. The value of waiting to gain more information on future price developments, and consequently on the optimal switching triggers, can be best viewed in a real options framework.

Conceptually, the switch between two volatile assets or commodities can be modelled as an exchange option. Margrabe (1978) and McDonald and Siegel (1986) model European and American perpetual exchange options, respectively, which are linear homogeneous in the underlying stochastic variables. Geltner, Riddiough and Stojanovic (1996) develop a framework for a perpetual option on the best of two underlying assets, applied to the case of two alternative uses for properties, and provide a comprehensive discussion of relevant assumptions for such a contingent-claims problem. Childs, Riddiough and Triantis (1996) extend this model to allow for redevelopment or switching between alternative uses.

The next section presents two real option models for an asset with switching opportunities between two outputs with uncertain prices, taking into account switching costs and operating costs. The first model is a quasi-analytical solution for multiple switching among the best of two outputs; the second for single one-way switching.

2 Output Switching

2.1 Assumptions

Consider a flexible facility which can be used to produce one of two different outputs by switching between operating modes. Assume the prices of the two outputs, x and y, are stochastic and possibly correlated and follow geometric Brownian motion (gBm):

$$dx = (\mu_x - \delta_x)x\,dt + \sigma_x\,x\,dz_x \tag{1}$$

$$dy = (\mu_y - \delta_y) y\, dt + \sigma_y\, y\, dz_y \qquad (2)$$

with the notations:

μ Required return on the output

δ Convenience (or asset) yield of the output

σ Volatility of the output

dz Wiener process (stochastic element)

The instantaneous cash flow in each operating mode is the respective price of the output less unit operating cost, assuming production of one (equivalent) unit per annum, $(x - c_x)$ in operating mode '1' and $(y - c_y)$ in operating mode '2'. The operating costs c_x and c_y are per unit (either hotel room or airline seat, or entire hotel, or aircraft). A switching cost of S_{12} is incurred when switching from operating mode '1' to '2', and S_{21} for switching back. The appropriate discount rate is r for non- stochastic elements, such as constant operating costs. For convenience and simplicity, assume that the appropriate discount rate for stochastic variables is δ and μ=r.

Further assumptions are that the operating costs are deterministic and constant, the lifetime of the asset is infinite, and the company is not restricted in the product mix choice because of selling commitments. Moreover, the typical assumptions of conventional real options theory apply, with interest rates, convenience yields, volatilities and correlation constant over time.

2.2 Quasi-analytical Solution for Multiple Switching

The asset value with opportunities to continuously switch between the two operating modes is given by the present value of perpetual cash flows in the current operating mode plus the option to switch to the alternative mode. Let V_1 be the asset value in operating mode '1', producing output x, and V_2 the asset value in operating mode '2', producing output y accordingly. The switching options depend on the two correlated stochastic variables x and y, and so do the asset value functions which are defined by the following partial differential equations:

(3)

$$\frac{1}{2}\sigma_X^2 x^2 \frac{\partial^2 V_1}{\partial x^2} + \frac{1}{2}\sigma_Y^2 y^2 \frac{\partial^2 V_1}{\partial y^2} + \rho\sigma_X\sigma_Y xy \frac{\partial^2 V_1}{\partial x \partial y} + (r - \delta_X)x\frac{\partial V_1}{\partial x} + (r - \delta_Y)y\frac{\partial V_1}{\partial y} - rV_1 + (x - c_X) = 0$$

$$\frac{1}{2}\sigma_X^2 x^2 \frac{\partial^2 V_2}{\partial x^2} + \frac{1}{2}\sigma_Y^2 y^2 \frac{\partial^2 V_2}{\partial y^2} + \rho\sigma_X\sigma_Y xy \frac{\partial^2 V_2}{\partial x \partial y} + (r-\delta_X)x\frac{\partial V_2}{\partial x} + (r-\delta_Y)y\frac{\partial V_2}{\partial y} - rV_2 + (y - c_y) = 0 \tag{4}$$

Two-factor problems which are linear homogeneous, i.e. $\mathrm{V}(\lambda \cdot \mathrm{x};\lambda \cdot \mathrm{y}) = \lambda \cdot \mathrm{V}(\mathrm{x};\mathrm{y})$, can typically be solved analytically by substitution of variables, so that the partial differential equation can be reduced to a one-factor differential equation. An example of this is the perpetual American exchange option in McDonald and Siegel (1986), or Paxson and Pinto (2005) considering competition. With constant switching costs, operating costs and multiple switching, the problem is no longer homogenous of degree one and the dimension reducing technique cannot be used.

Dockendorf and Paxson (2013) following Adkins and Paxson (2011) derive a quasi-analytical solution for a similar type of two-factor non-homogeneous problem. For two outputs, the partial differential equations are satisfied by the following general solutions:

$$V_1(x,y) = \frac{x}{\delta_x} - \frac{c_x}{r} + A x^{\beta_{11}} y^{\beta_{12}} \tag{5}$$

where β_{11} and β_{12} satisfy the characteristic root equation

$$\tfrac{1}{2}\sigma_x^2\beta_{11}(\beta_{11}-1) + \tfrac{1}{2}\sigma_y^2\beta_{12}(\beta_{12}-1) + \rho\sigma_x\sigma_y\beta_{11}\beta_{12} + \beta_{11}(r-\delta_x) + \beta_{12}(r-\delta_y) - r = 0, \tag{6}$$

and

$$V_2(x,y) = \frac{y}{\delta_y} - \frac{c_y}{r} + B x^{\beta_{21}} y^{\beta_{22}} \tag{7}$$

where β_{21} and β_{22} satisfy the characteristic root equation

$$\tfrac{1}{2}\sigma_x^2\beta_{21}(\beta_{21}-1) + \tfrac{1}{2}\sigma_y^2\beta_{22}(\beta_{22}-1) + \rho\sigma_x\sigma_y\beta_{21}\beta_{22} + \beta_{21}(r-\delta_x) + \beta_{22}(r-\delta_y) - r = 0 \tag{8}$$

Since the option to switch from x to y decreases with x and increases with y, β_{11} must be negative and β_{12} positive. Likewise, β_{21} must be positive and β_{22} negative. Switching between the operating modes always depends on the level of both x and y. At the switching points (x_{12}, y_{12}) and (x_{21}, y_{21}), the asset value in the current operating mode must be equal to the asset value in the alternative operating mode net of switching cost. These value matching conditions are:

$$A x_{12}^{\beta_{11}} y_{12}^{\beta_{12}} + \frac{x_{12}}{\delta_x} - \frac{c_x}{r} = B x_{12}^{\beta_{21}} y_{12}^{\beta_{22}} + \frac{y_{12}}{\delta_y} - \frac{c_y}{r} - S_{12} \tag{9}$$

$$A\,x_{21}^{\beta_{11}}\,y_{21}^{\beta_{12}}+\frac{x_{21}}{\delta_x}-\frac{c_x}{r}-S_{21}=B\,x_{21}^{\beta_{21}}\,y_{21}^{\beta_{22}}+\frac{y_{21}}{\delta_y}-\frac{c_y}{r} \tag{10}$$

Furthermore, smooth pasting conditions hold at the boundaries:

$$\beta_{11}Ax_{12}^{\beta_{11}-1}y_{12}^{\beta_{12}}+\frac{1}{\delta_x}=\beta_{21}Bx_{12}^{\beta_{21}-1}y_{12}^{\beta_{22}} \tag{11}$$

$$\beta_{12}\,A\,x_{12}^{\beta_{11}}\,y_{12}^{\beta_{12}-1}=\beta_{22}\,B\,x_{12}^{\beta_{21}}\,y_{12}^{\beta_{22}-1}+\frac{1}{\delta_y} \tag{12}$$

$$\beta_{11}\,A\,x_{21}^{\beta_{11}-1}\,y_{21}^{\beta_{12}}+\frac{1}{\delta_x}=\beta_{21}\,B\,x_{21}^{\beta_{21}-1}\,y_{21}^{\beta_{22}} \tag{13}$$

$$\beta_{12}\,A\,x_{21}^{\beta_{11}}\,y_{21}^{\beta_{12}-1}=\beta_{22}\,B\,x_{21}^{\beta_{21}}\,y_{21}^{\beta_{22}-1}+\frac{1}{\delta_y} \tag{14}$$

There are only 8 equations, (6) and (8) - (14), for 10 unknowns, β_{11}, β_{12}, β_{21}, β_{22}, A, B, x_{12}, y_{12}, x_{21}, y_{21}, so there is no completely analytical solution. Yet, for every value of x, there has to be a corresponding value of y when switching should occur, (x_{12}, y_{12}) from x to y and (x_{21}, y_{21}) from y to x. So a quasi-analytical solution can be found by assuming values for x, which then solves the set of simultaneous equations for all remaining variables, assuming $x = x_{12} = x_{21}$. This procedure is repeated for many values of x, providing the corresponding option values and the switching boundaries.

The switching thresholds suggested by the Marshallian rule are to switch from x to y when $\frac{y_{12}}{\delta_y}-\frac{x_{12}}{\delta_x}\geq S_{12}$ and from y to x when $-\frac{y_{21}}{\delta_y}+\frac{x_{21}}{\delta_x}\geq S_{21}$.

2.3 Quasi-analytical Solution for One-Way Switching

The solution for the asset value with a one-way switching option from the above model with continuous switching is straight-forward, so that the American perpetual option to switch from x to y (upgrading or reconfiguring a facility to accommodate upper-higher fare class) can be determined. The asset value V_{1S} is given by (5) with the characteristic root equation (18), and V_{2S} is given by (7) with B=0, thereby eliminating the option to switch back. Applying the same solution procedure as before, a quasi-analytical solution is obtained.

$$A x_{12}^{\beta_{11}} y_{12}^{\beta_{12}} + \frac{x_{12}}{\delta_x} - \frac{c_x}{r} = \frac{y_{12}}{\delta_y} - \frac{c_y}{r} - S_{12} \qquad (15)$$

Furthermore, smooth pasting conditions hold at the boundaries:

$$\beta_{11} A x_{12}^{\beta_{11}-1} y_{12}^{\beta_{12}} + \frac{1}{\delta_x} = 0 \qquad (16)$$

$$\beta_{12} A x_{12}^{\beta_{11}} y_{12}^{\beta_{12}-1} - \frac{1}{\delta_y} = 0 \qquad (17)$$

where β_{11} and β_{12} satisfy the characteristic root equation

$$\tfrac{1}{2}\sigma_x^2\beta_{11}(\beta_{11}-1) + \tfrac{1}{2}\sigma_y^2\beta_{12}(\beta_{12}-1) + \rho\sigma_x\sigma_y\beta_{11}\beta_{12} + \beta_{11}(r-\delta_x) + \beta_{12}(r-\delta_y) - r = 0, \qquad (18)$$

The characteristic root equation (18) together with value matching condition (15) and smooth pasting conditions (16) and (17) represents the system of 4 equations, while there are 5 unknowns, β_{11}, β_{12}, A, x_{12}, y_{12}. A solution is obtained if $x=x_{11}=x_{12}$, so $y_{12}/x_{12}=-\beta_{12}/\beta_{11}$, and $A=(-1/\delta_x)/(\beta_{11}*(x_{12}^{(\beta 11-1)})*y_{12}^{\beta 12})$.

3. Numerical Illustrations

Here are illustrative results for the multiple and single output switch models for an airline or entertainment/accommodation facility, assuming current operating costs for the upper and discount classes are the same in this case, as are the convenience yields, but the upper class fare volatility is double that for the discount class, and it is three times as expensive to reconfigure the facility for the upper class than for the discount class. Figure 1 shows that the option coefficients A and B are positive, β_{11} and β_{22} are negative and β_{12} and β_{21} are positive, thereby fulfilling the requirements from the theoretical model. The system of value matching conditions, smooth pasting conditions and characteristic root equations is fully satisfied.

The asset values are given in both operating modes, V_1 and V_2, and the level of y is indicated when it is optimal to switch from x to y (y_{12}) and vice versa (y_{21}). The asset value with no switching is lower currently for the discount class, but the asset value with the switching option is high. Figure 1 shows that when operating costs are 50, the asset value V_1 with continuous

Figure 1

	A	B	C	D	E	F	G	H
1	**Multiple American Perpetual Output Switch Option**							
3	OUTPUT x Discount Class	x	70					
4	OUTPUT y Upper Class	y	100					
5	Convenience yield of x	δ_x	0.04					
6	Convenience yield of y	δ_y	0.04					
7	Volatility of x	σ_x	0.20					
8	Volatility of y	σ_y	0.40					
9	Correlation x with y	ρ	0.00					
10	Risk-free interest rate	r	0.05					
11	Operating cost for x	c_x	50					
12	Operating cost for y	c_y	50					
15	Switching cost from x to y	S_{12}	300					
16	Switching cost from y to x	S_{21}	100					
17	PV of revenues x	X	750					
18	PV of revenues y	Y	1,500					
19	Switching boundary x to y	x_{12}	70					
20	Switching boundary y to x	x_{21}	70					
21		SOLUTION		OPTION	OPERATING			
22	Asset value in operating mode '1'	$V_1(x,y)$	2,216.01	1,466.01	750.00			
23	Asset value in operating mode '2'	$V_2(x,y)$	2,359.22	859.22	1,500.00			
24		A	8.37					
25		B	8.67					
26	Switching boundary x to y	y_{12} (x)	196.10					
27	Switching boundary y to x	y_{21} (x)	28.01					
28	Solution quadrant	β_{11}	-0.2144	*must be negative*				
29	Solution quadrant	β_{12}	1.3196	*must be positive*				
30	Solution quadrant	β_{21}	1.3828	*must be positive*				
31	Solution quadrant	β_{22}	-0.2775	*must be negative*				
32		EQUATIONS						
33	Value matching 1	EQ9	0.000					
34	Value matching 2	EQ10	0.000					
35	Smooth pasting 1A	EQ11	0.000					
36	Smooth pasting 1B	EQ12	0.000					
37	Smooth pasting 2A	EQ13	0.000					
38	Smooth pasting 2B	EQ14	0.000					
39	Solution quadrant 1	EQ6	0.000					
40	Solution quadrant 2	EQ8	0.000					
41		Sum	0.000					
42		SOLVER: SET C41=0, CHANGING C24:C31						
43	Marshall Thresholds	M x to y	82.00	C6*(C15+C3/C5)				
44	Marshall Thresholds	M y to x	66.00	C6*(-C16+C3/C5)				
45		M Spread	16.00	C48-C49				
46	EQ9	(C24*C19^C28*C26^C29+C19/C5-C11/C10-C25*C19^C30*C26^C31-C26/C6+C12/C10+C15)						
47	EQ10	(C24*C20^C28*C27^C29+C20/C5-C11/C10-C25*C20^C30*C27^C31-C27/C6+C12/C10-C16)						
48	EQ11	(C28*C24*C19^(C28-1)*C26^C29+1/C5-C30*C25*C19^(C30-1)*C26^C31)						
49	EQ12	(C29*C24*C19^C28*C26^(C29-1)-C31*C25*C19^C30*C26^(C31-1)-1/C6)						
50	EQ13	(C28*C24*C20^(C28-1)*C27^C29+1/C5-C30*C25*C20^(C30-1)*C27^C31)						
51	EQ14	(C29*C24*C20^C28*C27^(C29-1)-C31*C25*C20^C30*C27^(C31-1)-1/C6)						
52	EQ6	0.5*C7^2*C28*(C28-1)+0.5*C8^2*C29*(C29-1)+C9*C7*C8*C28*C29+C28*(C10-C5)+C29*(C10-C6)-C10						
53	EQ8	0.5*C7^2*C30*(C30-1)+0.5*C8^2*C31*(C31-1)+C9*C7*C8*C30*C31+C30*(C10-C5)+C31*(C10-C6)-C10						
54	SPREAD		168.09	C26-C27				
55	$V_1(x,y)$ EQ 5	C3/C5-C11/C10+C24*C3^C28*C4^C29						
56	$V_2(x,y)$ EQ 7	C4/C6-C12/C10+C25*C3^C30*C4^C31						

switching opportunities is valued at 2216 if the discount fare is x_{12}=70 with a volatility of 20%. The switching option value is the difference between the asset value and the value with no switching option, 2216-750=1466, and 2359-1500=859 for V_2. The option to switch between the two operating modes adds a lot to the inflexible asset value. Switching to output y is justified if y increases to 96% higher than currently, and back to x, if the y output price falls to almost one quarter of the current y price. The spread between y_{12} and y_{21} is due to switching costs and stochastic elements, and increases with high volatilities and low correlation, following real options theory. It should be noted that changing x also changes the switching boundaries y_{12} and y_{21}, and that the switching boundaries x_{12} and x_{21} for a given level of y can be determined in a similar way. The Jevons-Marshall rule of switching when the present value of the difference in the output values exceeds the switching costs indicates that a switch from x to y is justified now.

Figure 2

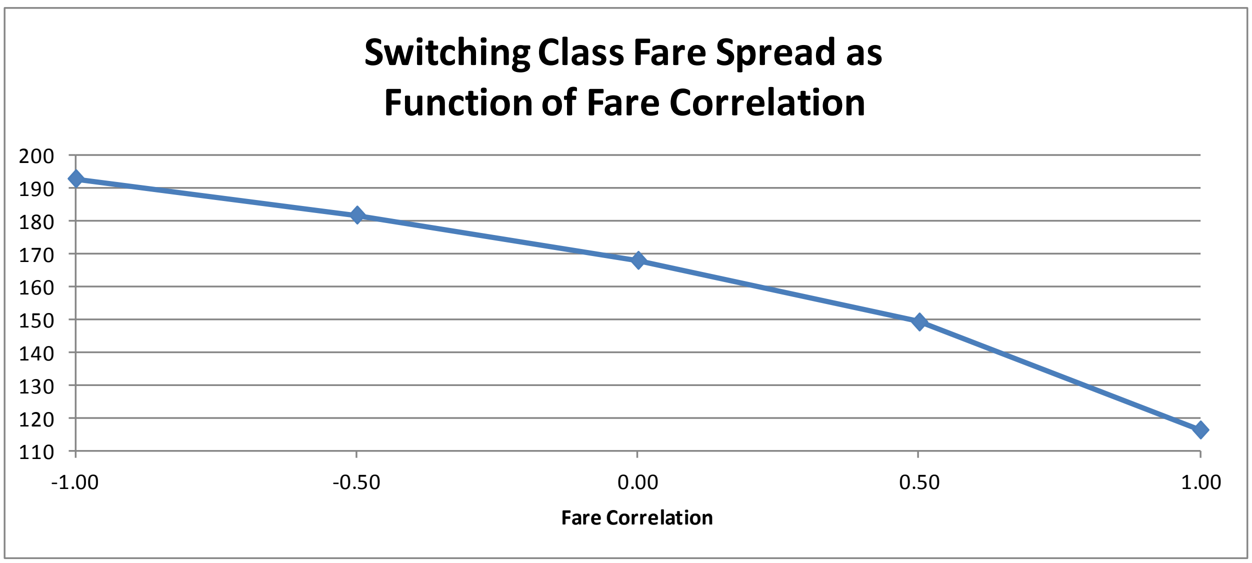


Figure 2 illustrates the sensitivity of the switching boundaries of the quasi-analytical solution for continuous switching to changes in x and y output price correlation. Switching boundaries are further apart when correlation is completely negative, and narrower when correlation is perfect, since exchange volatility decreases with increase of correlation. This is consistent with general real option theory because uncertainty is taken into account which delays switching in order to gain more information. In contrast, the Marshallian rule stipulates that switching is optimal as soon as the present value of expected cash flows after switching exceeds the present value of expected cash flows before switching by the switching cost regardless of the output volatilities or correlation.

Figure 3

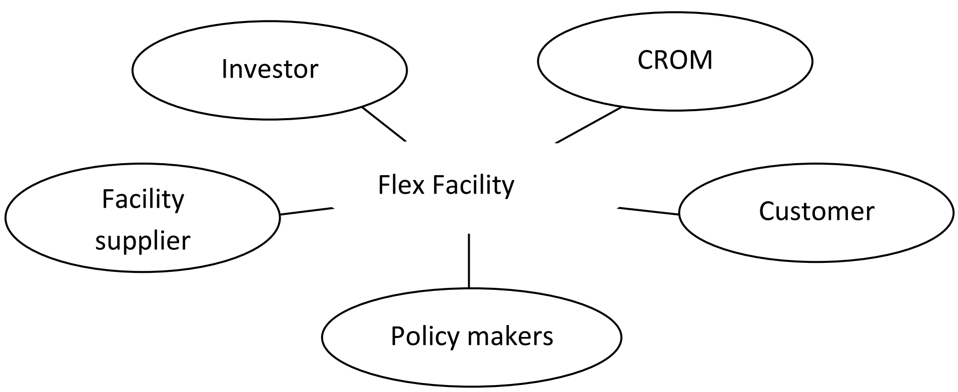
	A	B	C	D	E	F	G
1	**Continuous American Perpetual SINGLE SWITCH Option**						
2	ONE WAY SWITCH FROM OUTPUT x TO y						
3	OUTPUT x Discount Class	x	70				
4	OUTPUT y Upper Class	y	100				
5	Convenience yield of x	δ_x	0.04				
6	Convenience yield of y	δ_y	0.04				
7	Volatility of x	σ_x	0.20				
8	Volatility of y	σ_y	0.40				
9	Correlation x with y	ρ	0.00				
10	Risk-free interest rate	r	0.05				
11	Operating cost for x	c_x	50				
12	Operating cost for y	c_y	50				
13	Switching cost from x to y	S_{12}	300				
14							
15	PV of revenues x	X	750				
16	PV of revenues y	Y	1,500				
17	Switching boundary x to y	x_{12}	70				
18							
19		SOLUTION		OPTION VALUE			
20	Asset value in operating mode '1'	$V_1(x,y)$	2,044.51	1,294.51			
21	Asset value in operating mode '2'	$V_2(x,y)$	1,500.00	0			
22		A	9.54				
23	Switching boundary x to y	$y_{12}(x)$	344.55				
24	Solution quadrant	β_{11}	-0.2666	*must be negative*			
25	Solution quadrant	β_{12}	1.3123	*must be positive*			
26		EQUATIONS					
27	Value matching 1	EQ 15	0.000				
28	Smooth pasting 1A	EQ 16	0.000				
29	Smooth pasting 1B	EQ 17	0.000				
30	Q function	EQ 18	0.000				
31		Sum	0.000				
32		SOLVER: SET C31=0, CHANGING C22:C25					
33	EQ 15	(C22*C17^C24*C23^C25+C17/C5-C11/C10-C23/C6+C12/C10+C13)					
34	EQ 16	(C24*C22*C17^(C24-1)*C23^C25+1/C5)					
35	EQ 17	(C25*C22*C17^C24*C23^(C25-1)-1/C6)					
36	EQ 18	0.5*C7^2*C24*(C24-1)+0.5*C8^2*C25*(C25-1)+C9*C7*C8*C24*C25+C24*(C10-C5)+C25*(C10-C6)-C10					
37	SPREAD		**244.55**				
38	$V_1(x,y)$ EQ 5	C3/C5-C11/C10+C22*C3^C24*C4^C25					
39	$V_2(x,y)$ EQ 7	C4/C6-C12/C10					
40							
41		**CALCULATOR ANSWER**					
42		**$y_{12}(x)$**	**344.55**				
43			**-C25*C17/C24**				

When switching is only possible from x to y but not vice versa, the switching trigger y_{12S} is much (175%) higher as shown in Figure 3 because the decision cannot be reversed. The option value V_{1S} is 12% lower for switching from discount to upper class once only compared to multiple switching back and forth. Note that the value given by the "calculator answer" as specified in Exercise 13.1 is the same as the numerical answer provided by Solver.

4 Policy and Strategy Implications

There are a number of stakeholders shown in Figure 4 whose best decisions should be based on these switching models.

Figure 4



Investor

The real option value of these flexible facilities is substantially greater than the present value of current production (= inflexible facilities), at the current assumed input and output price levels. Note the focus of alert investors is on choosing the appropriate model and on forecasting input and output price volatilities and correlations. A myopic investment analyst using net present values will probably undervalue flexible facilities. Analysts may not have access to plant operating or switching costs, or indeed knowledge of any flexibility inherent in existing facilities, due conceivably to inadequate accounting disclosures, not currently required by accounting standard setting committees. Of course, realistic analysts may doubt that the chief option managers of flexible facilities will be aware of the potential optionality, or indeed make switches at appropriate times, so the Marshallian values might reflect a realistic allowance for management shortfalls.

Chief Real Options Manager

The alert chief real options manager ("CROM") is aware of output switching opportunities, the amount of switching costs, and periodically observes output prices, convenience yields (or proxies), updates expected volatilities and correlations, and so updates either Figure 1 or 3 appropriately. Observed current spreads between output prices are compared to the updated triggers for switching. Naturally part of the appropriate compensation for the CROM should be based on awareness of these opportunities, and performance in making actual output switches at appropriate times. Originally, the CROM would have calculated the value of a flexible facility V_1 or V_2, compared to an inflexible facility, which also indicates the warranted extra investment cost for facility flexibility. It would not be difficult to consider trade-offs for any deterministic lower efficiency due to the flexibility capacity.

Facility Supplier

Originally, suppliers of facilities to the CROM would have calculated the value of a flexible plant V_1 or V_2, compared to an inflexible facility, which also indicates the warranted extra investment price that could be charged for facility flexibility. With the illustrated parameter values, a hypothetical multiple switch facility is worth only some 5-15% more than a single switch plant, but much more than an inflexible facility. In designing flexible facilities, it would not be difficult to consider trade-offs for any lower efficiency due to the flexibility capacity against increased building costs.

Customers

Output customers may be aware of the limitations, or capacities, of producers to switch to higher price products, opportunistically. Other customers might seek long-term agreements mitigating the shifts in output prices implied in using real option approaches for operating flexible facilities.

Policy Makers

Taxpayers beware. There will be national airlines or treasured hotels without flexible facilities, or not aware of needing to reconfigure, as the economic environment changes. Those managers priced out of the market will seek government barriers for other operators, special tax relief, or input/output subsidies as conditions change.

III CAPACITY ALLOCATION BETWEEN FARE SEGMENTS

Suppose customers (for airline seats or hotel rooms) can be segmented into two distinct classes, luxury and discount, where perhaps purchase restrictions, refundability or servicing characterize each group (or use, such as business and leisure). Assume the same unit of capacity can be used to service both groups. The problem is to allocate the capacity among these two classes, if the discount class demand occurs first, by reserving a minimum number of capacity units for the luxury class,

Netessine and Shumsky (2002) define a booking limit b* as the maximum number of units that may be offered at the discount price, C as the number of units of capacity, and p* as the number of units reserved for the luxury class, where b*= C-p*.

As described in Phillips (2005), Littlewood (1972) proposed a simple rule that the optimal level allowed for discount booking at a fare p_d, when there are customers at a net full-fare p_f (having deducted any extra costs to service the full fare customers) and capacity is C, is the minimum of the inverse cumulative normal distribution function of the relative fares, or C.

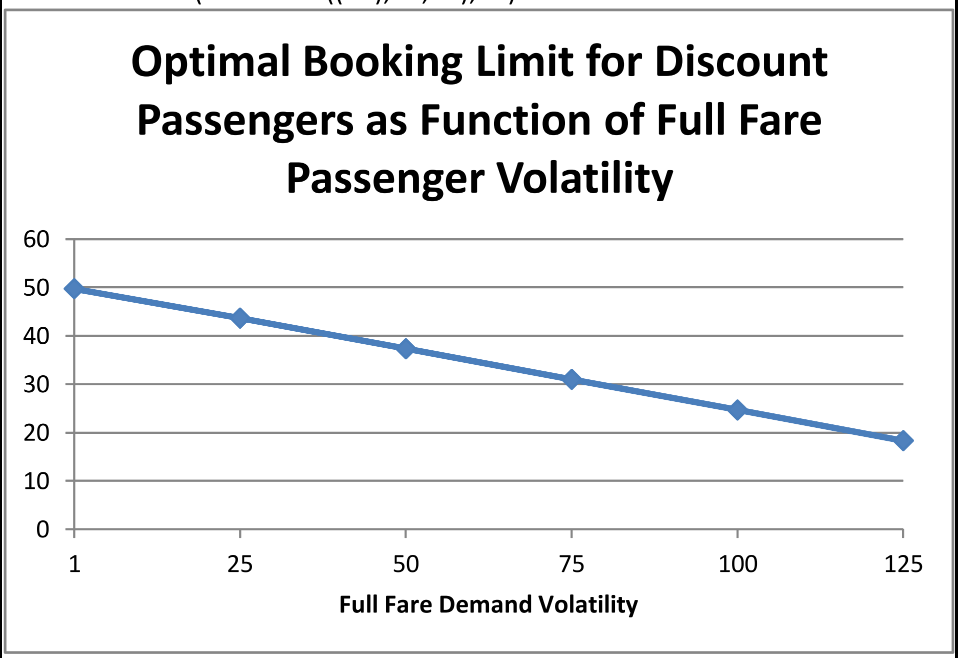
$$b^* = \min[F_d^{-1}(p_d / p_f), C] \tag{19}$$

The Excel formula for F_d^{-1} is NORMINV(p_d/p_f, mean, standard deviation), where "mean" is the average luxury fare demand for a particular flight (or hotel room for a particular date) and "standard deviation" is the volatility of that demand. Note that the mean and volatility of the discount fare demand is not relevant, nor is the capacity, except as a limit.

Phillips (2005) provides an example where the mean full-fare demand for a flight with a 100-seat aircraft is 50 with a standard deviation of 100. Suppose that the p_f=20 and p_d=8, the maximum bookings allowed for discount passengers should be 25. As shown in Figure 5, at very low full-fare volatility b* approaches the full-fare mean, and b* decreases with the increase of full-fare volatility. At high volatility it is worth taking the chance that full-fare passengers will eventually fill the plane, if the relative yield from the discount passengers is not high.

Figure 5

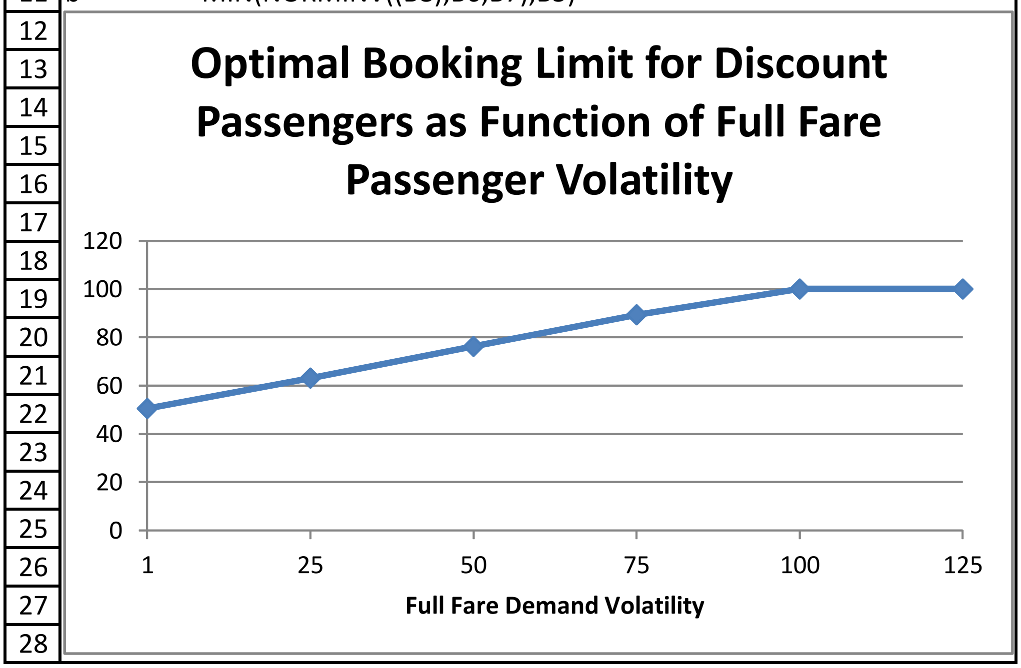
	A	B	C	D	E	F	G
1	**Stochastic Demand in Capacity Allocation**						
2	INPUT						
3	pf	$20	$20	$20	$20	$20	$20
4	pd	$8	$8	$8	$8	$8	$8
5	MAX C	100	100	100	100	100	100
6	D mean	50	50	50	50	50	50
7	D stdev	1	25	50	75	100	125
8	pd/pf	0.40	0.40	0.40	0.40	0.40	0.40
9	OUTPUT						
10	b*	50	44	37	31	25	18
11	b*	MIN(NORMINV((B8),B6,B7),B5)					



Note that when the discount fare is high relative to the full-fare, the optimal booking limit for the discount class increases as the full-fare volatility increases as shown in Figure 6, because large full-fare volatility implies also that the full-fare passengers may not fill the plane, which justifies allowing passengers at not-so-discounted fares to book seats in advance.

Figure 6

	A	B	C	D	E	F	G
1	**Stochastic Demand in Capacity Allocation**						
2	INPUT						
3	pf	$20	$20	$20	$20	$20	$20
4	pd	$14	$14	$14	$14	$14	$14
5	MAX C	100	100	100	100	100	100
6	D mean	50	50	50	50	50	50
7	D stdev	1	25	50	75	100	125
8	pd/pf	0.70	0.70	0.70	0.70	0.70	0.70
9	OUTPUT						
10	b*	51	63	76	89	100	100
11	b*	MIN(NORMINV((B8),B6,B7),B5)					

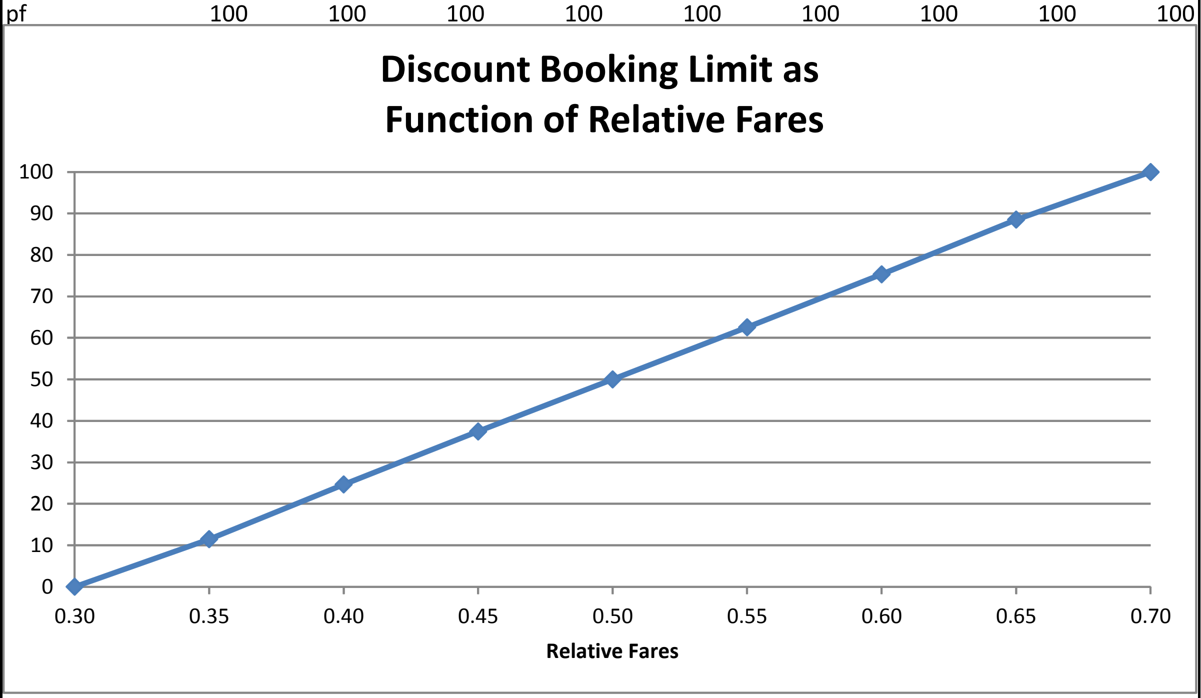


Another view of the effect of the relative fares on b* is shown in Table 7.1 of Phillips (2005).

The optimal booking limit b* increases almost linearly with the ratio of p_d/p_f as shown in Figure 7, ROW 32.

Figure 7

	A	B	C	D	E	F	G	H	I	J
1			Phillips Table 7.1							
2	**Book Limit**	0	11	25	37	50	63	75	89	100
3	PROB	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70
4	MEAN	50	50	50	50	50	50	50	50	50
5	STDEV	100	100	100	100	100	100	100	100	100
6	C	100	100	100	100	100	100	100	100	100
7	pd/pf	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**	**0.55**	**0.60**	**0.65**	**0.70**
8	pd	30	35	40	45	50	55	60	65	70
9	pf	100	100	100	100	100	100	100	100	100



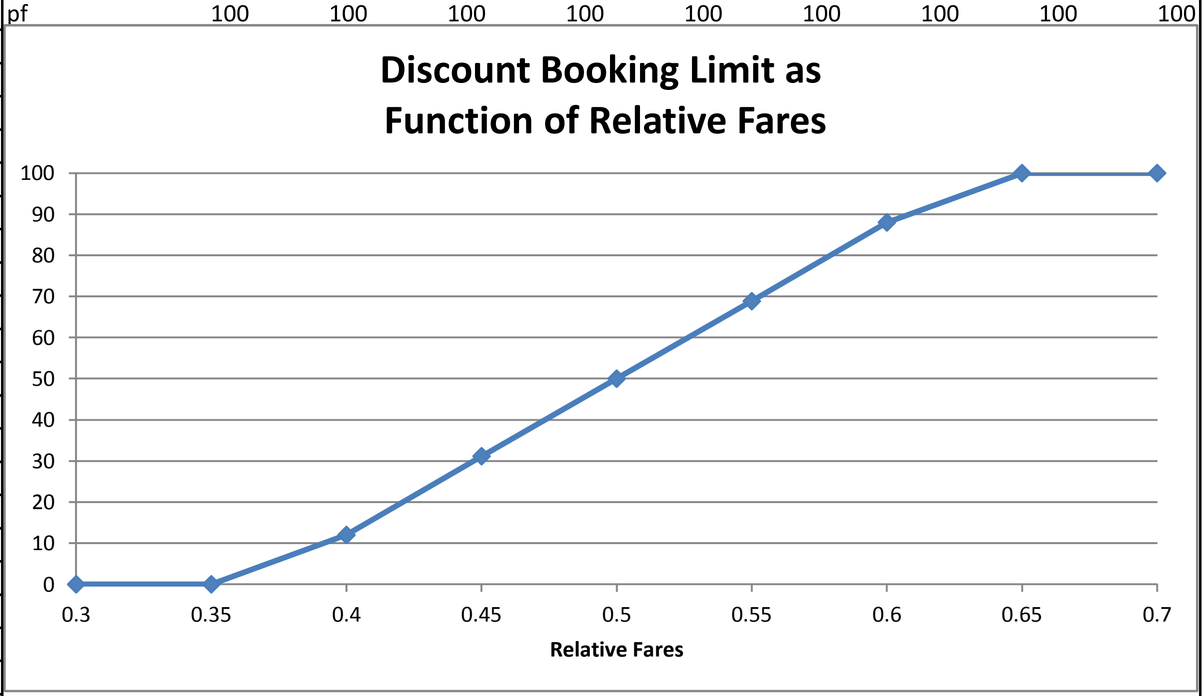
	A	B	C	D	E	F	G	H	I	J
30	STDEV	100								
31	Book Limit	MAX(MIN(NORMINV(B3,B4,B5),B6),0)								
32	Book Limit		12	25	37	50	63	75	88	101
33	Book Limit		-76.9+253.5*C7							

But at high full-fare volatility, b* reaches 100 (allocate all the available seats to the discount class if demanded) as the ratio of fares exceeds 65% (with these other parameter values) as shown in Figure 8.

If the ratio of fares is less than 35%, do not allow any discount fare bookings.

Figure 8

	A	B	C	D	E	F	G	H	I	J
1			Phillips Table 7.1							
2	**Book Limit**	0	0	12	31	50	69	88	100	100
3	PROB	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
4	MEAN	50	50	50	50	50	50	50	50	50
5	STDEV	150	150	150	150	150	150	150	150	150
6	C	100	100	100	100	100	100	100	100	100
7	pd/pf	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
8	pd	30	35	40	45	50	55	60	65	70
9	pf	100	100	100	100	100	100	100	100	100



	A	B	C
30	STDEV	150	
31	Book Limit	MAX(MIN(NORMINV(B3,B4,B5),B6),0)	

In summary, the ratio of the discount/full fares plus the mean and volatility of full fare demand determines the maximum of a limited capacity that should be allocated to discount fares. At high full fare demand volatility, the booking limit for discount fares is within a 35-65% range for these particular parameter values. At lower full fare demand volatility, the booking limit is approximately a linear function of the ratio of fares, the intercept and ratio coefficient dependent on the parameter values. The CROM for allocating limited capacity between market segments will update estimates for the mean and volatility of full fare demand periodically, changing also the fares charged for discount and full, and will update the maximum number of seats allowed for discount fares, dynamically.

IV OVERBOOKING OPTIONS

There are at least four types of reservation restrictions for airlines, hotels, restaurants, car rentals and other services: nonrefundable bookings, partially refundable, refundable advance payments, and reservations without advance payments (sometimes "guaranteed" by credit cards). Even nonrefundable customers may not show up ("no shows"= NS) for flights or room occupancy, so many service providers accept reservations (OB) in excess of capacity (C) which is termed "overbooking". Typically providers offer compensation (COM), or alternative arrangements, for those customers who are denied use at the time of departure in case C<=OB-NS.

Assume that all bookings are completely refundable, so an empty seat (underage penalty) is worth the ticket fare B, and that COM is paid to all customers denied boarding at departure. What is the optimal overbooking O* assuming COM is complete, that is there is no additional customer dissatisfaction that is not overcome by sufficient COM. As a classical newsvendor model, where the number of NS is normally distributed with mean μ and standard deviation σ, the smallest value O* is:

$$F(O^*) \geq \frac{B}{B + COM} = OB \tag{20}$$

To find O*, use NORMINV(OB, μ,σ). Netessine and Shumsky (2002) provide an example where the NS for a hotel with 210 rooms is normally distributed with μ=20 and σ=10, B=$105 and COM=$300 as shown in Figure 9. The optimal number of OB=14, so up to 224 bookings should be allowed.

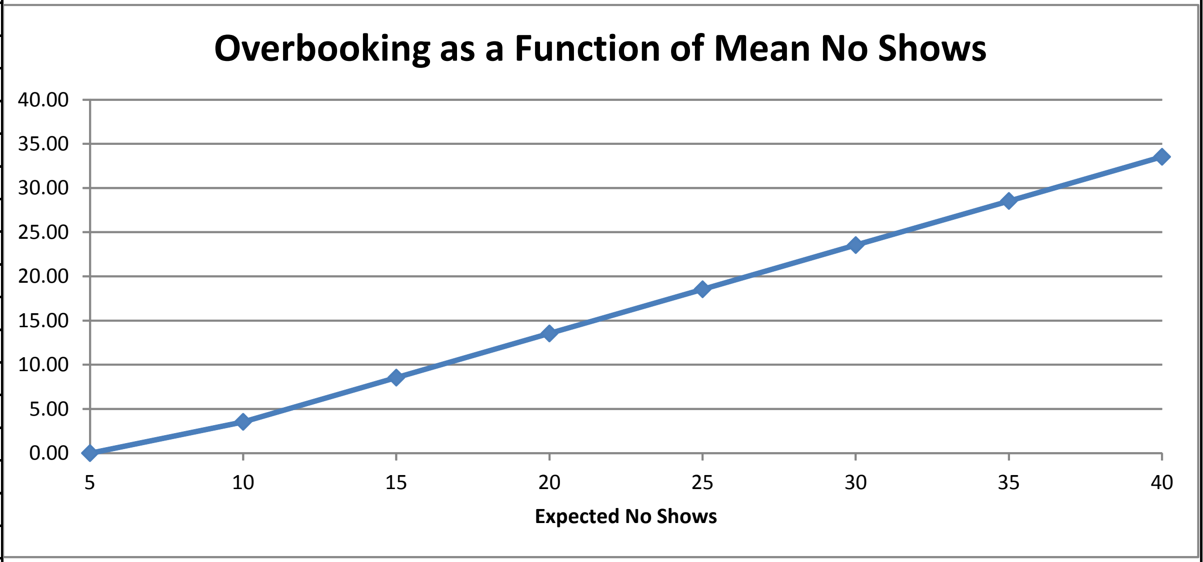
Figure 9

	A	B	C	D
1	**OPTIMAL OVERBOOKING with Stochastic No Shows**			
2	PRICE	105		
3	DENIED Compensation	300		
4	CAPACITY	210		
5	Expected NoShows	20		
6	Stdev NoShows	10.00		
7	NORMINV	13.54		
8	BOOKINGS ALLOWED	224		
9				
10	NORMINV	NORMINV((B2/(B2+B3)),B5,B6)		

How sensitive is the overbooking limit to variations in the estimated mean number of NoShows, volatility, and to the compensation required to satisfy bumped passengers? As expected, the number of optimal overbookings increases with the increase in the average number of NoShows as shown in Figure 10, so forecasting these expected NoShows is critical. Note the optimal OB is almost a linear function of the Expected NS with these parameter values.

Figure 10

	A	B	C	D	E	F	G	H	I
1				OPTIMAL OVERBOOKING with Stochastic No Shows					
2	PRICE	105	105	105	105	105	105	105	105
3	DENIED Compensation	300	300	300	300	300	300	300	300
4	CAPACITY	210	210	210	210	210	210	210	210
5	Expected NoShows	5	10	15	20	25	30	35	40
6	Stdev NoShows	10	10	10	10	10	10	10	10
7	OB NORMINV	0.00	3.54	8.54	13.54	18.54	23.54	28.54	33.54
8	BOOKINGS ALLOWED	210	214	219	224	229	234	239	244
9	NORMINV	MAX(NORMINV((B2/(B2+B3)),B5,B6),0)							
10	OB Approximate		3.55	8.55	13.55	18.55	23.55	28.55	33.55



28	OB Approximate		-6.45+C5

Optimal overbooking is sensitive to changes in the volatility of No Shows, but only if the compensation is high relative to the fare. If the volatility of the No Shows increases, the optimal overbooking does not decrease much (Figure 11) if the compensation is low, but decreases significantly if the compensation is high (Figure 12). So volatile customers should demand high compensation for being bumped to dampen the incentive of airlines to overbook aggressively.

Figure 11

	A	B	C	D	E	F	G	H	I
1				OPTIMAL OVERBOOKING with Stochastic No Shows					
2	PRICE	105	105	105	105	105	105	105	105
3	DENIED Compensation	110	110	110	110	110	110	110	110
4	CAPACITY	210	210	210	210	210	210	210	210
5	Expected NoShows	20	20	20	20	20	20	20	20
6	Stdev NoShows	1	5	10	15	20	25	30	35
7	NORMINV	19.97	19.85	19.71	19.56	19.42	19.27	19.13	18.98
8	BOOKINGS ALLOWED	230	230	230	230	229	229	229	229
9	NORMINV	MAX(NORMINV((B2/(B2+B3)),B5,B6),0)							

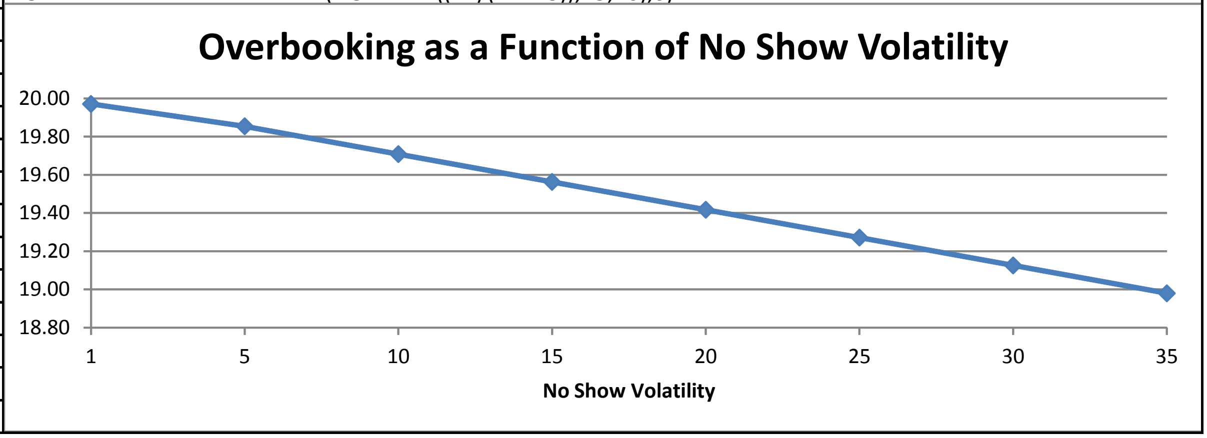
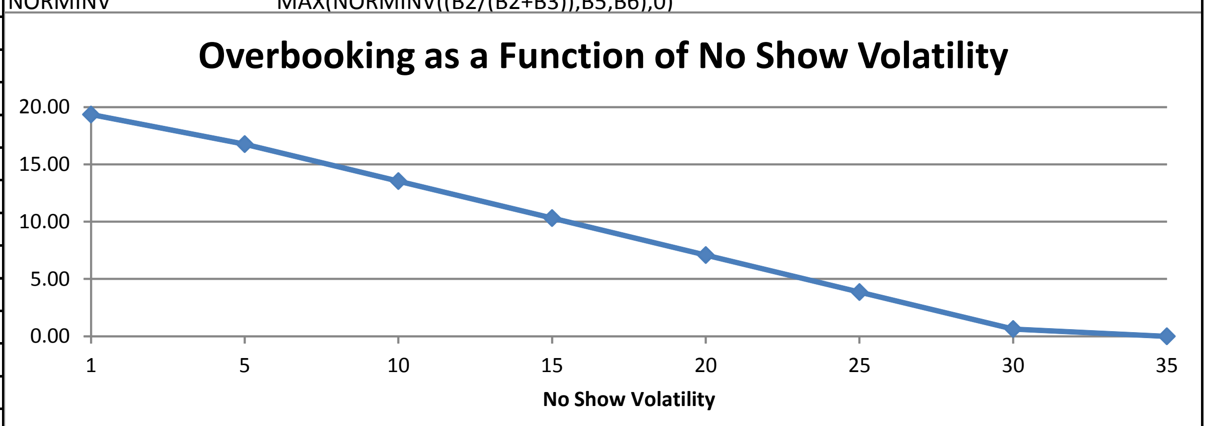


Figure 12

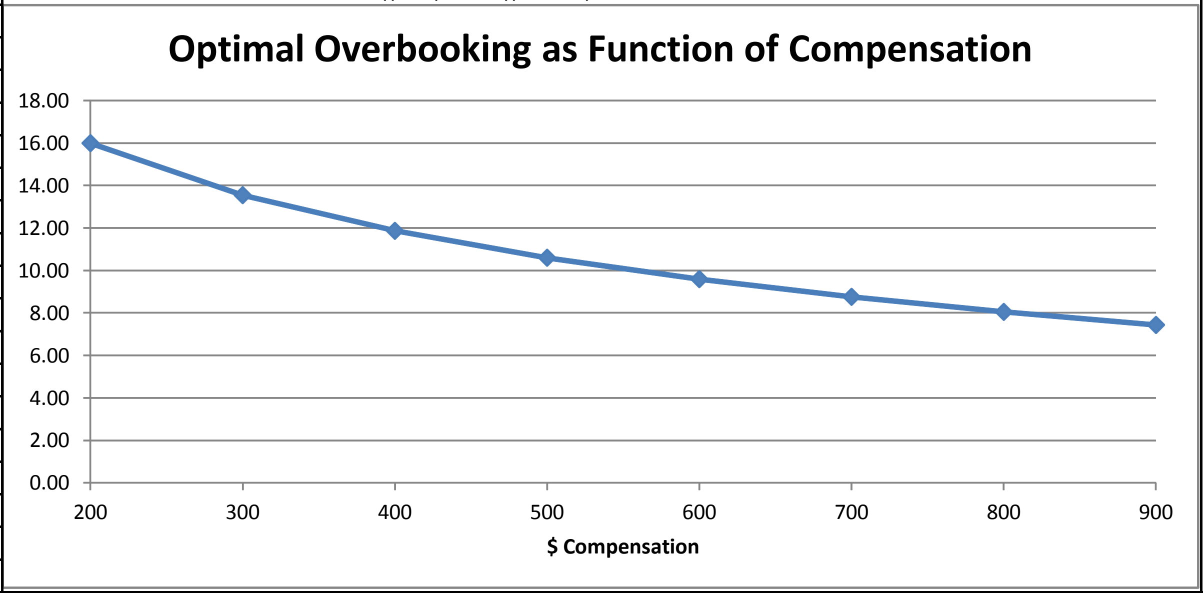
	A	B	C	D	E	F	G	H	I
1				OPTIMAL OVERBOOKING with Stochastic No Shows					
2	PRICE	105	105	105	105	105	105	105	105
3	DENIED Compensation	300	300	300	300	300	300	300	300
4	CAPACITY	210	210	210	210	210	210	210	210
5	Expected NoShows	20	20	20	20	20	20	20	20
6	Stdev NoShows	1	5	10	15	20	25	30	35
7	NORMINV	19.35	16.77	13.54	10.32	7.09	3.86	0.63	0.00
8	BOOKINGS ALLOWED	229	227	224	220	217	214	211	210
9	NORMINV	MAX(NORMINV((B2/(B2+B3)),B5,B6),0)							



This is re-emphasized in Figure 13 which shows that if the required compensation for bumped customers that show up becomes high, the optimal overbookings decline (but perhaps not by as much as expected, since increasing the COM by 450% results in the overbookings being reduced by half). Perhaps this indicates how much customers being bumped could demand in compensation.

Figure 13

	A	B	C	D	E	F	G	H	I
1				OPTIMAL OVERBOOKING with Stochastic No Shows					
2	PRICE	105	105	105	105	105	105	105	105
3	DENIED Compensation	200	300	400	500	600	700	800	900
4	CAPACITY	210	210	210	210	210	210	210	210
5	Expected NoShows	20	20	20	20	20	20	20	20
6	Stdev NoShows	10	10	10	10	10	10	10	10
7	NORMINV	15.99	13.54	11.86	10.60	9.59	8.76	8.05	7.44
8	BOOKINGS ALLOWED	226	224	222	221	220	219	218	217
9	NORMINV	NORMINV((B2/(B2+B3)),B5,B6)							



V OTHER REAL REVENUE OPTIONS

Netessine and Shumsky (2002) and the textbooks by Phillips (2005) and Talluri and van Ryzin (2004) discuss several other real revenue options. <u>Demand forecasting</u> models calibrate different distributions to historical demand, and provide various forecasting improvements as customer preferences change daily, seasonally and over time. Perhaps GARCH models here are appropriate. <u>Dynamic booking limits</u> allow for new information that appears over time, such as revised demand for luxury seats nearing departure. Capacity variations allow for expansion of existing capacity as demand evolves, such as rental car firms obtaining additional cars from nearby associated units (or from other rental firms in the same location) as demand increases. Nonlinear overbooking compensation options include capacity expansion (especially rental cars) as a form of avoided compensation, and also compensation that decreases if the NS suddenly decline. Buy-ups and cannibalizations allow for discount passengers beyond the booking limit to reserve luxury seats (buy-ups) (even last minute upgrades) and upper class passengers to purchase discount seats (cannibalizations). Variety (among and within) booking classes allows different durations for room reservations within a class, and for group (or convention) bookings where additional facilities are required or a minimum number of rooms are guaranteed. Wholly and partially refundable fares offer a menu of possible options with restrictions, with stochastic rebates and allowance. Some of these issues can be incorporated into the simple market segmentation and switching models; some require novel analysis, including numerical solutions.

Several journals contain useful, if sometimes complex, discussions, including the INFORM library (*Management Science, Marketing Science, Decision Science, Transportation Science,* and *The Journal of Revenue and Pricing Management*), the *European Journal of Operational Research*, and the *Journal of Economic Dynamics & Control*.

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EXERCISE 13.1

Dom Paulo Hotels is negotiating with Smith Luxury Holidays and also Jones Discount Enjoyments (current long term client) for booking arrangements next year. Jones promises to pay 50=x per room, while Smith offers to pay 100=y if the hotel installs certain amenities such as a grand fitness room and 25 meter pool which cost 300, a once-for-all upgrade of the facility. It would cost 70 to service the Smith customers, but only 10 for the Jones lot. Both room rates are volatile (20%), are 50% correlated, yield 4%, and the current interest rate is 5%. Dom Paulo calculates that β_{11}=-.4505, β_{12}=1.991, y_{12}/x_{12}=-β_{12}/β_{11}, and A=(-1/δ_x)/(β_{11}*($x_{12}^{(\beta 11-1)}$)* $y_{12}^{\beta 12}$). Should Dom switch now to Smith? If not, what is the opportunity to switch sometime worth, RHS of EQ 5?

EXERCISE 13.2

MassAir has just received permission to fly directly between Manchester and Boston, currently not serviced by any airlines, and hopes to attract both business and discount passengers, using Boeing 737s with a seat capacity of 100. Business fares via Dublin with AerLinqus are $900, and discount fares via JFK with NorwegianAir are $500 (including BOS-JFK connections). From previous experience on similar routes it appears the business class mean will be 50, with a standard deviation of 50, and the additional cost of servicing business customers with lobsters, wine and massage is $100, which is about the same as the inconvenience of stopping over in DUB. What is the maximum number of discount bookings with two weeks in advance that should be allowed? What if lobsters-wine-massage cost MassAir $150? F_d^{-1} (p_d/p_f) is approximately 200*(p_d/p_f)-60 for these parameter values.

EXERCISE 13.3

MassAir has decided to offer just discount fares for their direct flights between Manchester and Boston, using Boeing 737s with a seat capacity of 200. The discount fare is comparable to NorwegianAir of $300. From a year’s experience it appears discount class mean No Shows will be 10, with a standard deviation of 10. Norwegian Air offers a compensation of $700 for bumped passengers due to overbooking. What optimal bookings should MassAir allow? OB is approximately equal to -5.24 +μ_{NS} . What if μ_{NS} =40?

PROBLEM 13.4

Dom Paulo Hotels is negotiating with Smith Luxury Holidays and also Jones Discount Enjoyments (current long term client) for booking arrangements next year. Jones promises to pay 50=x per room, while Smith offers to pay 100=y if the hotel installs certain amenities such as redecorated rooms, room service and a sports massage facility which cost 300, but which could be reversed at a cost of 100 for Discount customers. It cost 30 to service both the Smith and Jones customers. Both room rates are volatile (20%), are 50% correlated, yield 4%, and the current interest rate is 5%. Should Dom switch now to Smith? If not, what is the opportunity to switch back and forth worth?

PROBLEM 13.5

After offering direct flights between Manchester and Boston with a seat capacity of 100, MassAir wants to experiment with increasing business fares since there is still no other direct flight between these two centers of education. Business fares have been $1000, and discount fares $500. From the year's experience the business class mean is 50, with a standard deviation of 100, and the additional cost of servicing business customers with lobsters, wine and massage is $100. What is the maximum number of discount bookings that should be allowed?

What is the total expected revenue if all of the discount fares are booked up to the booking limit, and the average business booking is 50? If business fares are in the range of $1000 to $1900 and if the mean business fare booking is reduced by β*Fare Increase, and the standard deviation is increased by (β/2)*Fare Increase, where β=.0075, should MassAir remain a mixed-budget or a luxury only airline?

PROBLEM 13.6

MassAir has decided to offer just discount fares for their direct flights between Manchester and Boston, using Boeing 737s with a seat capacity of 100. The discount fare is comparable to NorwegianAir of $500 (including BOS-JFK connections). From a year's experience it appears discount class mean No Shows will be 50, with a standard deviation of 100. MassAir has been offering only $500 compensation for bumped passengers due to overbooking. Over the past few months a large number of customers have complained that the compensation offer (less the cost of the next day's flight) will hardly cover a room at the Omni Parker Boston hotel plus dinner at the Union Oyster House. Members of both the Cabot and Lodge families have recently been bumped, which they consider unacceptable. Why have so many passengers been bumped? What compensation might be required by the Mass Transportation Authority to virtually eliminate this onerous practice?